Tractability results for structured quantified CNF-formulas via knowledge compilation

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Abstract. We show how knowledge compilation can be used as a tool for QBF. More precisely, we show that one can apply quantification on certain data structures used in knowledge compilation which in combination with the fact that restricted classes of CNF-formulas can be compiled into these data structures can be used to show fixed-parameter tractable results for QBF.

It is well known that restricting the interaction between variables and clauses in CNF-formulas makes several hard problems tractable. For example, SAT and #SAT can be solved in time $2^{O(k)}|F|$ when F is a CNF formula with a primal graph of treewidth k [Sze04,SS10]. Many extensions of this result have been shown these last ten years for more general graph measures such as modular treewidth or clique width [PSS16,SS13,BCM15,STV14]. In [BCMS15], Bova et al. recently explained these results using Knowledge Compilation, a subarea of artificial intelligence that systematically studies and compares the properties of different representations for knowledge: many classes of structured CNF can be represented by small Boolean circuits known as structured deterministic DNNF (sd-DNNF). Such circuits have strong restrictions making several problems such as satisfiability and model counting on them tractable.

In this ongoing work, we show how these circuit representations can be used in the context of quantification. To this end, we give a simple algorithm that, given an sd-DNNF C and a subset X of variables, outputs an sd-DNNF C' computing $\exists XC$. We show a similar result to construct an sd-DNNF C' computing $\forall XC$. In general, the size of C' blows up exponentially during our transformation and this is unavoidable since there are strong exponential lower bounds in the setting. But here we show this exponential blowup in fact depends only on the *width* of the input circuit and not on the size. Since many structured CNF-formulas, such as those of bounded treewidth, can be translated into bounded width sd-DNNF, we are able to construct sd-DNNF for the quantified formula where the blowup is relatively tame in our setting which yields fixed-parameter tractable algorithms for several problems. Figure 1 depicts the overall scheme that we use to construct such algorithms.

For instance, our algorithm can be used to show that the number of models of a partially quantified CNF-formula F of treewidth k with t blocks of quantifiers can be computed in time $2^{\cdot^{\cdot^{\cdot^{2^{O(k)}}}}}|F|^2$ with $t+1$ exponentiations. This generalises

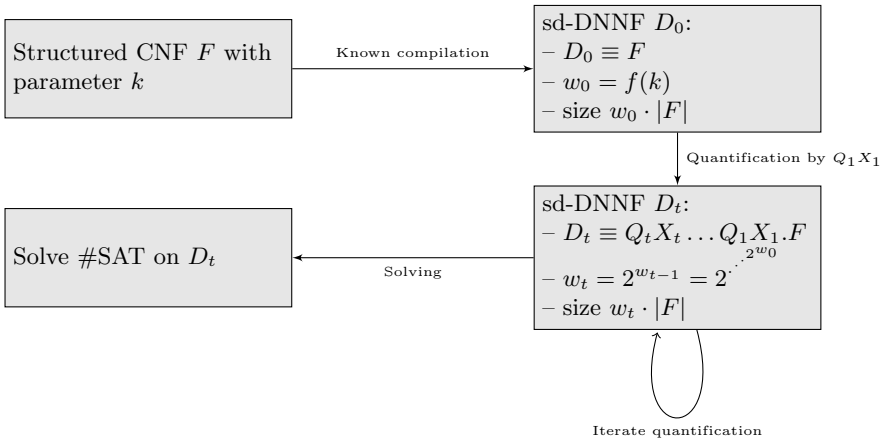

Fig. 1. The overall scheme for proving tractability results on structured quantified CNF.

a result by Chen [Che04] where the fixed-parameter tractability of QBF on such formulas was shown with a comparable complexity. Moreover, our algorithm also applies to more general width measures.

We complement our algorithm with lower bounds that show that our construction is essentially optimal in several respects.

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