Embedding Intuitionistic Modal Logics in Higher-Order Logic. An Easy Task?

Talk Abstract for the Second Women in Logic (WiL) Workshop

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1 Motivation

Intuitionistic logics have shown to constitute the foundations for many practical applications. For example, they serve as basis for state-of-the art proof assistents like Coq. Nevertheless, they have also been controversially discussed. David Hilbert wrote that: "Taking the principle of excluded middle from the mathematician would be the same, say, as proscribing the telescope to the astronomer or to the boxer the use of his fists."[4] Indeed, this property of intuitionistic logic gives rise to plenty of new challenges. When trying to embed intuitionistic logics in higher-order logic, monotonicity has to be minded. Furthermore, it is desirable to prove a set of frame conditions equivalent to the axioms used to build different intuitionistic logics. This talk will give a short overview on the challenges occurring due to the absence of the principle of excluded middle and discuss possible solutions.

2 Content

First, a short overview about modal intuitionistic logic (IML) is given. This is important as two systems of this logic are popular, intuitionistic IK and constructive IK [1]. We will focus on intuitionistic IK and introduce important properties, like monotonicity. Following this, the embedding of multi-modal logics into higher-order logic(HOL) which has been realised by Christoph Benzmüller et al. in [2] is presented.

To show the motivation of an embedding from IML to HOL the example of the intuitionistic modal logic cube is introduced. This cube is constituted by different intuitionistic logics which are built when different subsets of the intuitionistic modal logic axioms T,D,B,4,5 are added to IK. After embedding IML in HOL, connections between different logics can be verified automatically using the proof assistant Isabelle/HOL [3]. Because of the absence of duality between the $\Box$ and $\Diamond$ operator the intuitionistic modal logic axioms are much more complicated than in the multi-modal case. This makes the embedding and verification process much more complex.

Subsequently, a proposal of an embedding is presented. Then, the verification process of the intuitionistic modal logic cube is shown. Finally, an example is given why the presented approach is still not fully sufficient and why a different definition of monotonicity is needed. It is shown that the principle of excluded middle can be proven using this definition and possible reasons for this are discussed.

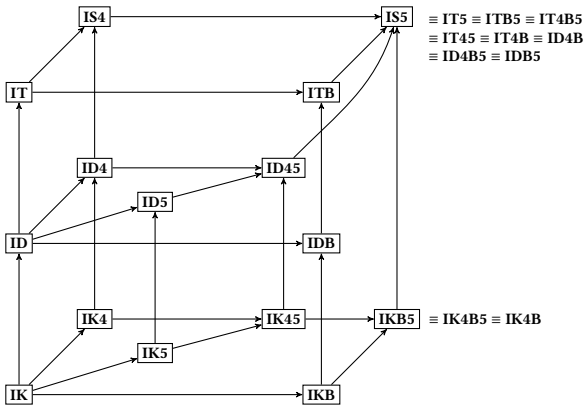

Figure 1. The intuitionistic modal logic cube

3 Conclusion

The talk presents our ongoing research on the intuitionistic model logic cube and the limitations of a first embedding approach of IML in HOL using Isabelle/HOL. In future work, we plan to replace the current definition of monotonicity so that it does not imply the principle of excluded middle. As an embedding into higher-order logic would also allow for other applications and not only the verification of the IML cube, the future work on this topic is promising.

References

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